
# Chapter 3
# CNC Math

## Objectives

*Information in this chapter will enable you to:*

- Identify various geometric shapes.
- Apply various geometric principles to solve problems.
- Solve right triangle unknowns.
- Apply trigonometric principles to determine coordinate values.

## Technical Terms

acute angle
adjacent angles
angle
arc
bisect
chord
circle
circumference
complementary angles
congruent
cosecant
cosine
cotangent
diagonal
diameter
equilateral triangle
function
hypotenuse
isosceles triangle
obtuse angle
parallel
parallelogram
perpendicular
polygons
proposition
Pythagorean theorem
quadrilateral
radius
rectangle
reflex angle
right angle
right triangle
scalene triangle
secant
segment
sine
square
straight angle
supplementary angles
tangent
transversal
triangle
trigonometric functions
trigonometry
vertex

## Geometric Terms

The following is a list of geometric terms and their definitions. These terms will be used throughout this chapter and the remainder of this textbook. Study them before continuing.





- ***Bisect.*** To divide into two equal parts.
- ***Congruent.*** Having the same size and shape.
- ***Diagonal.*** Running from one corner of a four-sided figure to the opposite corner.
- ***Parallel.*** Lying in the same direction but always the same distance apart.
- ***Perpendicular.*** At a right angle to a line or surface.
- ***Segment.*** That part of a straight line included between two points.
- ***Tangent.*** A line contacting a circle at one point.
- ***Transversal.*** A line that intersects two or more lines.

## Angles

An ***angle*** (∠) is the figure formed by the meeting of two lines at the same point or origin called the ***vertex***. See **Figure 3-1**. Angles are measured in degrees (°), minutes (′), and seconds (″). A degree is equal to 1/360 of a circle, a minute is equal to 1/60 of 1°, and a second is equal to 1/60 of 1′.

There are many types of angles, **Figure 3-2**. An ***acute angle*** is greater than 0° and less than 90°. An ***obtuse angle*** is greater than 90° and less than 180°. A ***right angle*** is exactly 90°. A ***straight angle*** is exactly 180°, or a straight line. A ***reflex angle*** is greater than 180° and less than 360°.

An angle can also be described by its relationship to another angle. See **Figure 3-3**. ***Adjacent angles*** are two angles that use a common side. ***Complementary angles*** are two angles that equal 90°. ***Supplementary angles*** are two angles that equal 180°, or a straight line.

## Polygons

***Polygons*** are figures with many sides that are formed by line segments. Polygons are named according to the number of sides and angles they have. For example, a decagon is a polygon with ten sides; *deca* comes from the Latin word for ten.

### Triangles

A ***triangle*** is a three-sided polygon. There are a number of types of triangles, **Figure 3-4**. A ***right triangle*** has a 90° (right) angle. An ***isosceles triangle*** has two equal sides and two equal angles. An ***equilateral triangle*** has three equal sides; all angles are equal (60°). A ***scalene triangle*** has three unequal sides and unequal angles.

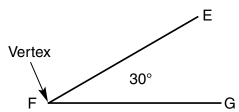



**Figure 3-1.** Angle defined as EFG.

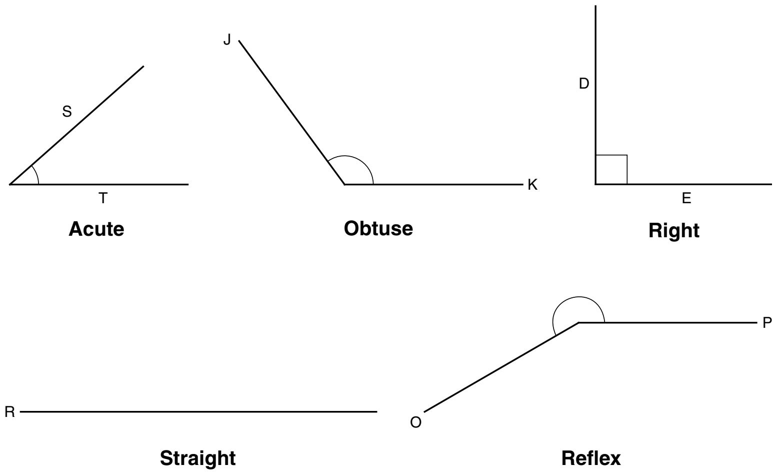



**Figure 3-2.** Various types of angles.

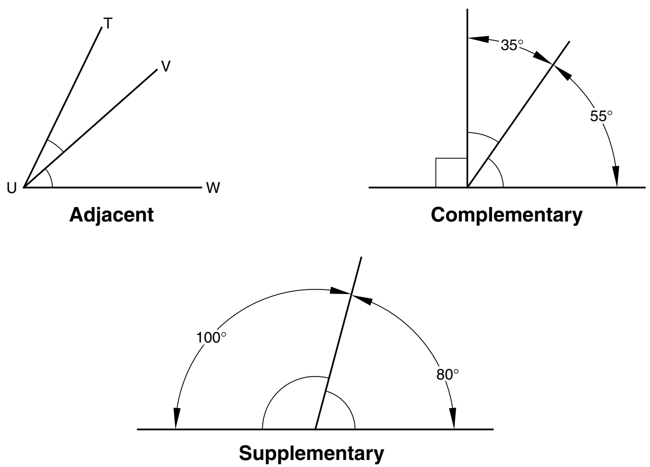



**Figure 3-3.** Describing an angle in relationship to another angle.



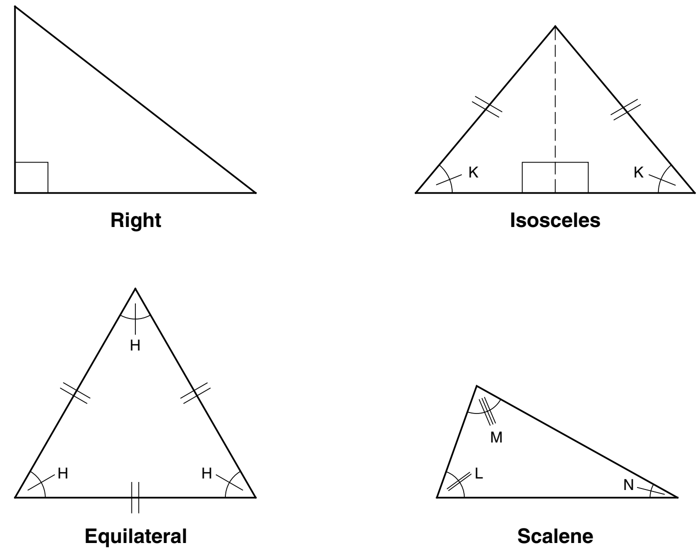



**Figure 3-4.** Examples of various types of triangles. The sum of the angles in a triangle always equals 180°.

## Quadrilaterals

A ***quadrilateral*** is a polygon with four sides, **Figure 3-5**. A line drawn from one angle (intersecting corner) of a quadrilateral to the opposite angle is called a *diagonal*.

### Square

A ***square*** has four equal sides and four right (90°) angles. The opposite sides of a square are parallel to each other. The diagonals of a square bisect the four angles and each other. The diagonals are equal and perpendicular to each other. The diagonals form congruent angles (equal in size and shape).

### Rectangle

A ***rectangle*** is a quadrilateral with equal opposite sides and four right angles. The opposite sides are parallel to each other. The diagonals are equal, bisect each other, and create two pairs of congruent triangles.

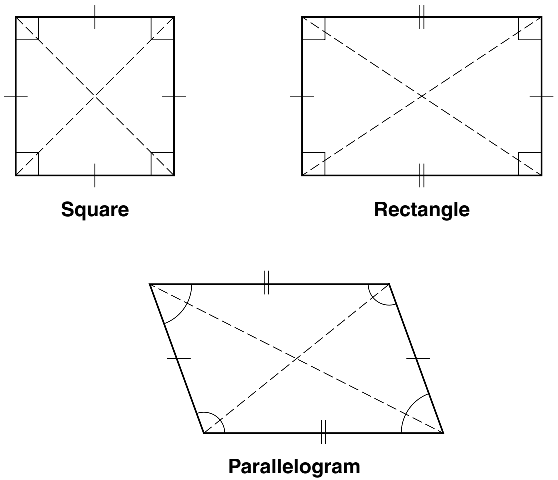



**Figure 3-5.** The three types of quadrilaterals. Quadrilaterals have four interior angles that total 360°.

### Parallelogram

A ***parallelogram*** is a quadrilateral with equal opposite sides and equal opposite angles. The diagonals bisect each other and create two pairs of congruent triangles.

## Circles

A ***circle*** is a set of points, located on a plane, that are equidistant from a common central point (center point). See **Figure 3-6**. There are a number of terms that are used to describe various aspects of a circle, **Figure 3-7**.

The ***diameter*** is the segment that connects two points on a circle and intersects through the center of the circle. The *size* of a circle is its diameter.

The ***radius*** is a segment that joins the circle center to a point on the circle circumference. Radius has half the value of diameter.

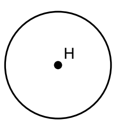



**Figure 3-6.** All points defining a circle are equidistant from the center point.



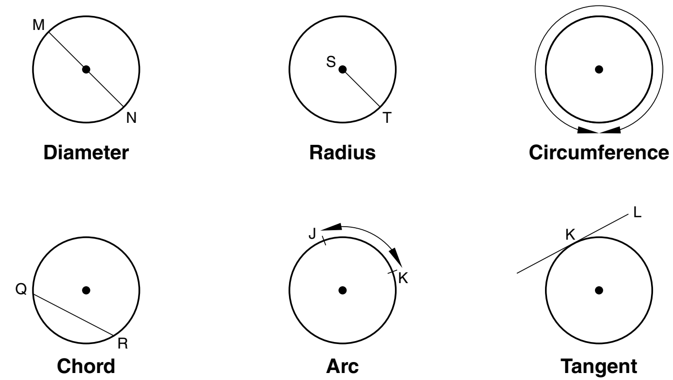



**Figure 3-7.** Illustrations of various terms relating to a circle.

The ***circumference*** is the distance around a circle.

A ***chord*** is a segment that joins any two points on the circumference of a circle.

An ***arc*** is a curved portion of a circle.

A ***tangent*** to a circle is a line that intersects a circle at a single point. For example, as shown in Figure 3-7, Line L is tangent to the circle and intersects the circle at Point K.

## Propositions

A ***proposition*** is a statement to be proved, explained, or discussed. Following are a number of geometric propositions.

- **Opposite angles are equal.** When two lines intersect, they form equal angles. Thus, in **Figure 3-8**, Angle 1 equals Angle 3, and Angle 2 equals Angle 4.
- **Two angles are equal if they have parallel corresponding sides.** Thus, in **Figure 3-9**, Angle 1 equals Angle 2.
- **A line perpendicular to one of two parallel lines is perpendicular to the other line.** Thus, in **Figure 3-10**, Lines R and S are perpendicular to Line T.

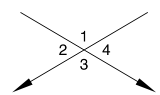



**Figure 3-8.** Two intersecting lines form four angles with the opposite angles being equal.

- **Alternate interior angles are equal.** If two parallel lines are intersected by a third line (transversal), then alternate interior angles are equal to each other. Thus, in **Figure 3-11**, Angle 3 equals Angle 6, and Angle 4 equals Angle 5.
- **Alternate exterior angles are equal.** When two parallel lines are intersected by a third line (transversal), then alternate exterior angles are equal to each other. Thus, in **Figure 3-11**, Angle 1 equals Angle 8, and Angle 2 equals Angle 7.
- **Corresponding angles are equal.** When two parallel lines are intersected by a third line (transversal), then all corresponding angles are equal. Thus, in **Figure 3-11**, Angle 1 equals Angle 5, Angle 3 equals Angle 7, Angle 2 equals Angle 6, and Angle 4 equals Angle 8.
- **The sum of the interior angles of a triangle is 180°.** Thus, in **Figure 3-12**, Angle 1 plus Angle 2 plus Angle 3 equals 180°.
- **The exterior angle of a triangle is equal to the sum of the two nonadjacent interior angles.** Thus, in **Figure 3-13**, Angle 4 equals Angle 1 plus Angle 2.

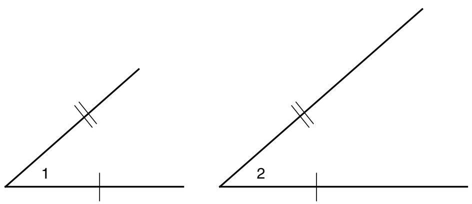



**Figure 3-9.** Two angles with corresponding parallel sides are equal.

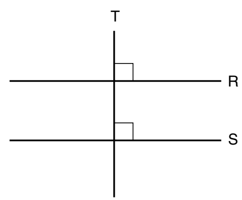



**Figure 3-10.** A transversal line perpendicular to one parallel line is perpendicular to the other parallel line. Lines R and S are parallel.

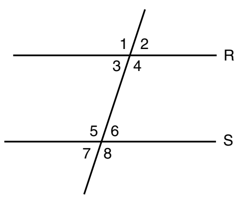



**Figure 3-11.** Two parallel lines intersected by a third line form alternate angles that are equal to each other. Interior Angles 4 and 5 are equal along with 3 and 6. Exterior Angles 1 and 8 are equal along with 2 and 7.

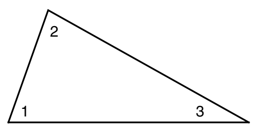



**Figure 3-12.** Angles 1, 2, and 3 form a triangle totaling 180°.



- **Two angles having their corresponding sides perpendicular are equal.** Thus, in **Figure 3-14**, Angle 1 equals Angle 2.
- **A line taken from a point of tangency to the center of a circle is perpendicular to the tangent.** Thus, in **Figure 3-15**, Line TW is perpendicular to Line UV.
- **Two tangent lines drawn to a circle from the same exterior point cause the corresponding segments to be equal in length.** Thus, in **Figure 3-16**, Segment ML equals Segment MN.

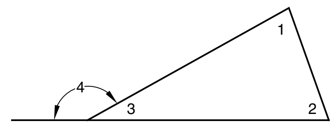



**Figure 3-13.** Angle 4, an exterior angle, equals the sum of Angles 1 and 2, which are nonadjacent interior angles.

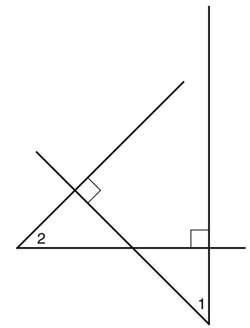



**Figure 3-14.** Angle 1 is equal to Angle 2 because their corresponding sides are perpendicular to each other.

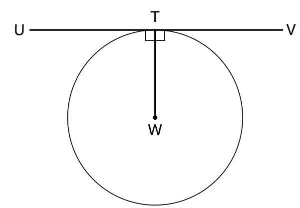



**Figure 3-15.** Line TW, developed from the tangency point to the circle's center, is perpendicular to tangent Line UV.

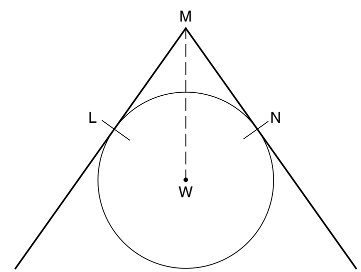



**Figure 3-16.** Lines MN and ML are tangent to the circle and share an endpoint and are, therefore, equal in length.

# Trigonometry

***Trigonometry*** is the area of mathematics that deals with the relationship between the sides and angles of a triangle. Triangles are measured to find the length of a side (leg) or to find the number of degrees in an angle. In CNC machining, trigonometry is used to determine tool location relative to part geometry.

Trigonometry deals with the solution of triangles, primarily the right triangle. See **Figure 3-17**. A right triangle has one angle that is 90° (Angle c), and the sum of all angles equals 180°. Angles a and b are acute angles, which means they each are less than 90°. Angles a and b are complementary angles, which means they total 90° when added.

The three sides of a triangle are called the hypotenuse, side opposite, and side adjacent. Side C is called the ***hypotenuse***, because it is opposite the right angle. It always is the longest side.

Sides A and B are either opposite to or adjacent to either of the acute angles. It depends on which acute angle is being considered. Side A is the side opposite Angle a, but is the side adjacent to Angle b. Side B is the side opposite Angle b, but is the side adjacent to Angle a. For example, when referring to Angle b, Side A is adjacent and Side B is opposite. Or, when referring to Angle a, Side B is adjacent and Side A is opposite.

As stated earlier in this chapter, angles are usually measured in degrees, minutes, and seconds, **Figure 3-18**. There are 360° in a circle, 60′ in a degree, and 60″ in a minute. As an example, 31 degrees, 16 minutes, and 42 seconds is written as 31°16′42″. Angles can also be given in decimal degrees, such as 34.1618 (34°9′42″).

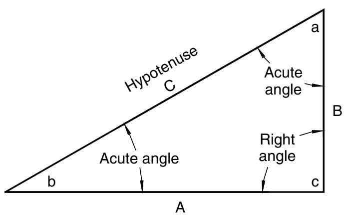



**Figure 3-17.** Lines are labeled as capital letters and angles are labeled as small letters. Note that Line A is opposite Angle a, Line B is opposite Angle b, and Line C is opposite Angle c.

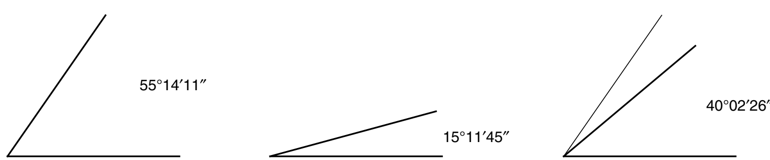



**Figure 3-18.** Illustrations of various angles containing degrees, minutes, and seconds.



Angles can be added by aligning the degrees, minutes, and seconds and adding each column separately. When totals for the minutes or seconds columns add up to 60 or more, subtract 60 (or 120, if appropriate) from that column, then add 1 (or 2, if appropriate) to the next column to the left (the higher column).

| | | |
|---|---|---|
| 16° | 33′ | 14″ |
| 5° | 17′ | 16″ |
| 38° | 55′ | 49″ |
| 59° | 105′ | 79″ |
| | **-60** | **-60** |
| | 45′ | 19″ |
| **+1** | **+1** | |
| 60° | 46′ | 19″ |

In the example, the total is 59°105′79″ when the angles are added. Since 79″ equals 1′19″ and 105′ equals 1°45′, the final answer is 60°46′19″.

When subtracting angles, place the degrees, minutes, and seconds under each other and subtract the separate columns. If not enough minutes or seconds exist in the upper number of a column, then borrow 60 from the next column to the left of it and add it to the insufficient number.

| | | | | | | |
|---|---|---|---|---|---|---|
| 55° | 14′ | 11″ | **borrow 60″→** | 55° | 13′ | 71″ |
| –15° | 11′ | 45″ | | –15° | 11′ | 45″ |
| | | | | 40° | 2′ | 26″ |

Since 11″ is smaller than 45″, 60″ must be borrowed from 14′. When 15°11′45″ is subtracted from 55°13′71″, the final answer is 40°2′26″.

## Using Trigonometry

Trigonometry is the most valuable mathematical tool used by a programmer for calculating cutter or tool nose locations. Trigonometric functions are absolute values derived from the relationships existing between angles and sides of a right triangle. A ***function*** is a magnitude (size or dimension) that depends upon another magnitude. For example, a circle's circumference is a function of its radius, since the circle size depends on the extent of its radius value.

In the triangle shown in **Figure 3-19**, A/B is the ratio of two sides and therefore a function of Angle d. As Angle d increases to the dashed line, the function will change from A/B to E/B. This shows that the ratio of two sides of a triangle depends on the size of the angles of the triangle.

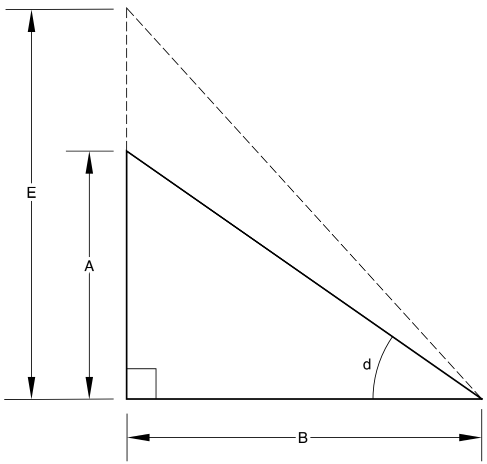



**Figure 3-19.** As Line A increases in length to become Line E, Angle d increases in value, when Line B remains the same.

## Trigonometric Functions

Since there are three sides (legs) to a triangle, there exist six different ratios of sides. These ratios are the six ***trigonometric functions*** of sine, cosine, tangent, cotangent, secant, and cosecant. Each ratio is named from its relationship to one of the acute angle in a right triangle. The right angle is never used in calculating functions. A function is obtained by dividing the length of one side by the length of one of the other sides. These functions can be found in math texts and many references, such as *Machinery's Handbook*. Special books also exist that give primarily trigonometric tables and values. In addition, many calculators can compute trigonometric values. **Figure 3-20** is a partial table of trigonometric functions covering 33°. To find the cosine of 33°58′, read down the minute column to 58 minutes, then read across the row. Under the column labeled cosine, you find value 0.82936, which is cosine 33°58′.



**Trigonometric Functions for Angles**

| Angle | Sine | Cosine | Tangent | Cotangent | Secant | Cosecant |
|---|---|---|---|---|---|---|
| 33°0′ | 0.54464 | 0.83867 | 0.64941 | 1.53986 | 1.19236 | 1.83608 |
| 33°1′ | 0.54488 | 0.83851 | 0.64982 | 1.53888 | 1.19259 | 1.83526 |
| 33°2′ | 0.54513 | 0.83835 | 0.65024 | 1.53791 | 1.19281 | 1.83444 |
| 33°3′ | 0.54537 | 0.83819 | 0.65065 | 1.53693 | 1.19304 | 1.83362 |
| 33°4′ | 0.54561 | 0.83804 | 0.65106 | 1.53595 | 1.19327 | 1.83280 |
| 33°5′ | 0.54586 | 0.83788 | 0.65148 | 1.53497 | 1.19349 | 1.83198 |
| 33°6′ | 0.54610 | 0.83772 | 0.65189 | 1.53400 | 1.19372 | 1.83116 |
| 33°7′ | 0.54635 | 0.83756 | 0.65231 | 1.53302 | 1.19394 | 1.83034 |
| 33°8′ | 0.54659 | 0.83740 | 0.65272 | 1.53205 | 1.19417 | 1.82953 |
| 33°9′ | 0.54683 | 0.83724 | 0.65314 | 1.53107 | 1.19440 | 1.82871 |
| 33°10′ | 0.54708 | 0.83708 | 0.65355 | 1.53010 | 1.19463 | 1.82790 |
| 33°11′ | 0.54732 | 0.83692 | 0.65397 | 1.52913 | 1.19485 | 1.82709 |
| 33°12′ | 0.54756 | 0.83676 | 0.65438 | 1.52816 | 1.19508 | 1.82627 |
| 33°49′ | 0.55654 | 0.83082 | 0.66986 | 1.49284 | 1.20363 | 1.79682 |
| 33°50′ | 0.55678 | 0.83066 | 0.67028 | 1.49190 | 1.20386 | 1.79604 |
| 33°51′ | 0.55702 | 0.83050 | 0.67071 | 1.49097 | 1.20410 | 1.79527 |
| 33°52′ | 0.55726 | 0.83034 | 0.67113 | 1.49003 | 1.20433 | 1.79449 |
| 33°53′ | 0.55750 | 0.83017 | 0.67155 | 1.48909 | 1.20457 | 1.79371 |
| 33°54′ | 0.55775 | 0.83001 | 0.67197 | 1.48816 | 1.20480 | 1.79293 |
| 33°55′ | 0.55799 | 0.82985 | 0.67239 | 1.48722 | 1.20504 | 1.79216 |
| 33°56′ | 0.55823 | 0.82969 | 0.67282 | 1.48629 | 1.20527 | 1.79138 |
| 33°57′ | 0.55847 | 0.82953 | 0.67324 | 1.48536 | 1.20551 | 1.79061 |
| 33°58′ | 0.55871 | 0.82936 | 0.67366 | 1.48442 | 1.20575 | 1.78984 |
| 33°59′ | 0.55895 | 0.82920 | 0.67409 | 1.48349 | 1.20598 | 1.78906 |

**Figure 3-20.** Partial table showing values of the six trigonometric functions sine, cosine, tangent, cotangent, secant, and cosecant as they relate to 33° and various minutes.

The six trigonometric functions are defined relative to the relationships between two sides of the right triangle, **Figure 3-21**. These relationships are:

- ***Sine* (sin).** The ratio of the opposite side to the hypotenuse.

$$\text{Sine a} = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{A}{C}$$

- ***Cosine* (Cos).** The ratio of the adjacent side to the hypotenuse.

$$\text{Cosine a} = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{B}{C}$$

- ***Tangent* (Tan).** The ratio of the opposite side to the adjacent side.

$$\text{Tangent a} = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{A}{B}$$

- ***Cotangent* (Cot).** The ratio of the adjacent side to the opposite side. It is the reciprocal of the tangent function.

$$\text{Cotangent a} = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{B}{A}$$

- ***Secant* (Sec).** The ratio of the hypotenuse to the adjacent side. It is the reciprocal of the cosine function.

$$\text{Secant a} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{C}{B}$$

- ***Cosecant* (Csc).** The ratio of the hypotenuse to the opposite side. It is the reciprocal of the sine function.

$$\text{Cosecant a} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{C}{A}$$

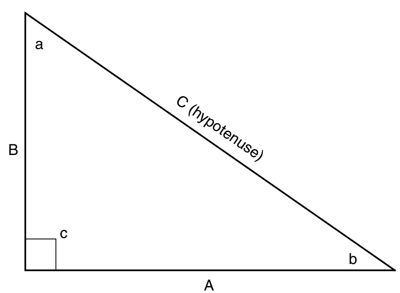



**Figure 3-21.** The hypotenuse is always the longest side (leg) of a right triangle.



The six functions given are related to Angle a, but also can be applied to Angle b as well. Therefore, Sin b = B/C, Cos b = A/C, etc., shows that any function of Angle a is equal to the cofunction of Angle b. From that relationship, the following are derived:

- sin a = A/C = cos b
- cos a = B/C = sin b
- tan a = A/B = cot b
- cot a = B/A = tan b
- sec a = C/B = csc b
- csc a = C/A = sec b

With Angle a and Angle b being complementary, the function of any angle is equal to the cofunction of its complementary angle. Therefore, sin 70° = cos 20°, and tan 60° = cot 30°.

## Working with Triangles

In programming, an individual will be working with various applications of radii, such as cutter radius, arc radius, circle radius, and corner radius. At times, the radius the programmer works with will appear like the triangle in **Figure 3-22**, where the long leg of the triangle is the radius. At other times, the triangle will appear like the triangle in **Figure 3-23**, where a leg will be the radius. When dealing with triangles, a programmer must recognize the configuration being worked with. There

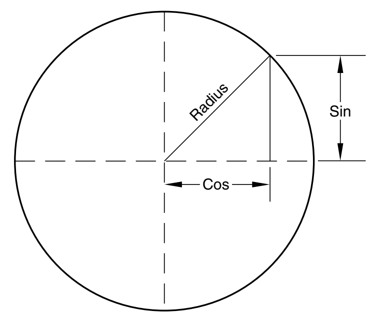



**Figure 3-22.** Programmers will sometimes use the radius value as the hypotenuse when determining location values.

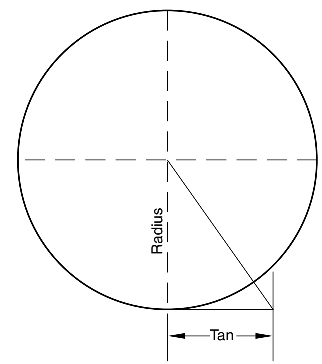
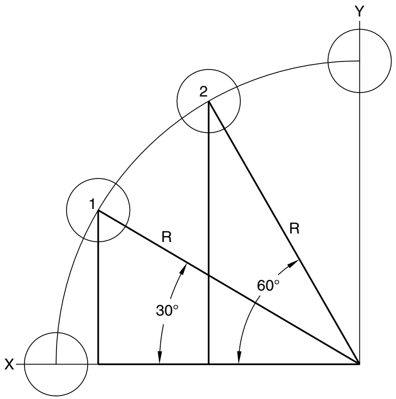


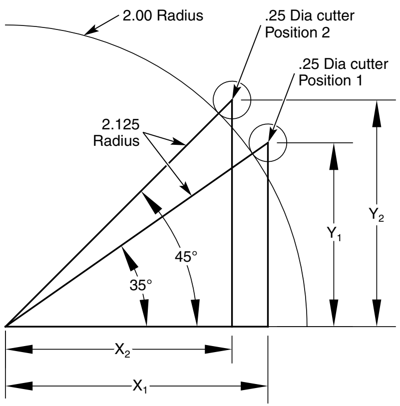

**Figure 3-23.** Programmers may use the radius value as a leg to solve the other missing values of the right triangle.

are a number of applications where a programmer will use the radius and triangle to determine distances. In **Figure 3-24**, the radius and triangle are applied to determine a bolt circle, tool path, and intersection. In **Figure 3-25**, the radius and triangle are applied to determine a cutter path.




| Position 1 | Position 2 |
|---|---|
| R = 1.5″ | R = 1.5″ |
| Angle from the horizontal axis = 30° | Angle from the horizontal axis = 60° |
| Y = sin 30 (1.5″) | Y = sin 60 (1.5″) |
| Y = 0.5 (1.5″) | Y = 0.866 (1.5″) |
| Y = 0.75″ | Y = 1.299″ |
| X = cos 30 (1.5″) | X = cos 60 (1.5″) |
| X = 0.866 (1.5″) | X = 0.5 (1.5″) |
| X = 1.299″ | X = 0.75″ |

**Figure 3-24.** Using a radius value and the construction of a right triangle to determine hole locations. Note: Angle values are used to solve remaining leg values.




| Position 1 | Position 2 |
|---|---|
| R = 2.0″ | R = 2.0″ |
| Cutter diameter = 0.250″ | Cutter diameter = 0.250″ |
| Angle from the horizontal axis = 35″ | Angle from the horizontal axis = 45″ |
| $Y_1$ = sin 35 (2.125″) | $Y_2$ = sin 45 (2.125″) |
| $Y_1$ = 0.5735 (2.125″) | $Y_2$ = 0.707 (2.125″) |
| $Y_1$ = 1.2186″ | $Y_2$ = 1.502″ |
| $X_1$ = cos 35 (2.125″) | $X_2$ = cos 45 (2.125″) |
| $X_1$ = 0.819 (2.125″) | $X_2$ = 0.707 (2.125″) |
| $X_1$ = 1.740″ | $X_2$ = 1.502″ |

**Figure 3-25.** Using trigonometry to calculate tool locations.



There are a number of situations where triangles can be applied to determine a cutter path. To plot a cutter path, the cutter radius is added or subtracted from the part outline. The cutter path is the path in which the centerline of the spindle moves along the plane, staying away from the part by the amount of the tool radius. To cut a 90° corner, the cutter moves past the edges of the part a distance equal to the cutter radius. See **Figure 3-26**. To cut an acute (less than 90°) angle, the cutter moves past the corner of the workpiece equal to the distances represented by the X dimension of the shaded triangle in **Figure 3-27**. To cut an obtuse (greater than 90°) angle, as shown in **Figure 3-28**, the same formula is used. Notice that the distance the cutter has to travel beyond the end of the part is greater than the cutter radius for acute angles and less than the cutter radius for obtuse angles.

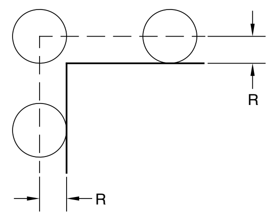
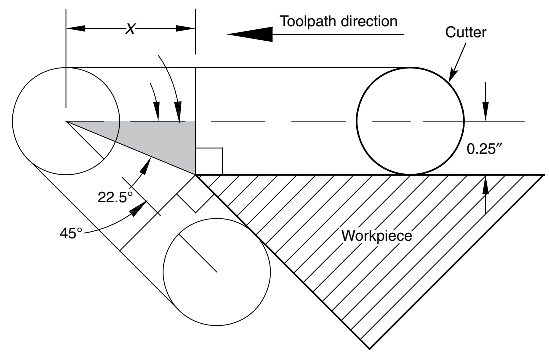



**Figure 3-26.** The location of a cutter when it is about to make a 90° move is shown. The radius of the cutter is used when determining the cutter location from the corner of the workpiece.




Angle on workpiece is 45°

Cutter radius is 0.25″

$$\tan 22.5° \text{ (half of 45°)} = \frac{0.25''}{X}$$

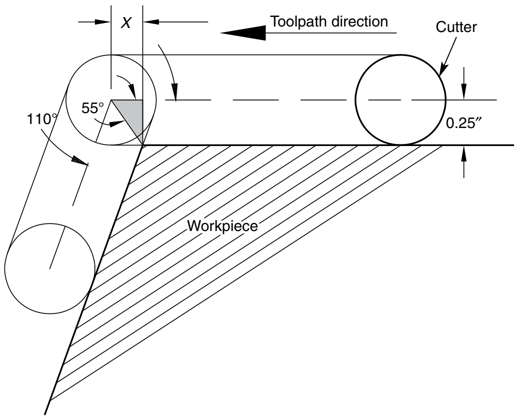
$$X = \frac{0.25''}{\tan 22.5°}$$

$$X = \frac{0.25''}{0.414}$$

$$X = 0.604''$$

The cutter must travel 0.604″ past the end of the workpiece to begin cutting the bottom-left side.

**Figure 3-27.** Illustration showing the triangle that must be solved to calculate the position of a cutter when cutting an acute angle on a workpiece.




Angle on workpiece is 110°

Cutter radius is 0.25″

$$\tan 55° \text{ (half of 110°)} = \frac{0.25''}{X}$$

$$X = \frac{0.25''}{\tan 55°}$$

$$X = \frac{0.25''}{1.428}$$

$$X = 0.175''$$

The cutter must travel 0.175″ past the end of the workpiece to begin cutting the left side.

**Figure 3-28.** The triangle that must be solved to calculate the position of the end mill when cutting an obtuse angle on a workpiece.

## Pythagorean Theorem

The ***Pythagorean theorem*** states a special relationship that exists among the three sides of a right triangle. It states that *the length of the hypotenuse squared equals the sum of the squares of the other two side lengths.* So, if the lengths of any two sides of a right triangle are given, the length of the third side can be calculated by using the Pythagorean theorem:

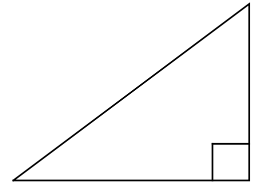
$$A^2 + B^2 = C^2$$

In **Figure 3-29**, Side C is equal to 5 and Side B is equal to 3. The value for A (the third side of the triangle) can be determined by using the formula $C^2 = A^2 + B^2$. To solve for A, substitute the known values into the formula to get $5^2 = A^2 + 3^2$, then square the values to get $25 = A^2 + 9$. Next, isolate the unknown variable by subtracting 9 from both sides of the equation to get $16 = A^2$. Finally, take the square root of both sides of the equation, to get $4 = A$. So, the length of Side A is 4.

To cut a 90° rounded corner on a workpiece, we can use the Pythagorean theorem to plot the toolpath of the cutter. See **Figure 3-30**. The radius on the workpiece is 1″. The cutter diameter is 0.25″ (0.125″ radius).

To cut partial arcs, we can use a combination of trigonometric functions and the Pythagorean theorem to plot the positions of the cutter. See **Figure 3-31**.

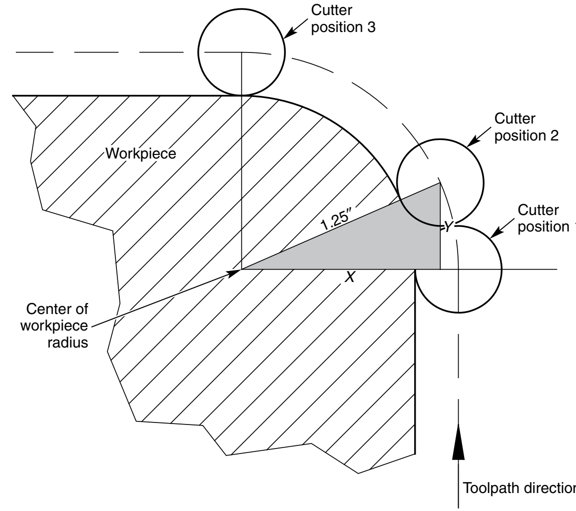




Pythagorean theorem:

$$C^2 = A^2 + B^2$$
$$5^2 = A^2 + 3^2$$
$$25 = A^2 + 9$$
$$25 - 9 = A^2$$
$$\sqrt{16} = \sqrt{A^2}$$
$$4 = A$$

**Figure 3-29.** Cutter locations when cutting a 90° radius corner on a workpiece.




The distance from the center of the workpiece radius to the center of the cutter at Position 1 is 1.25″ (workpiece radius + cutter radius). Therefore, the formula to find the X dimension for each change in Y is $X^2 + Y^2 = 1.25^2$.

$$X^2 + Y^2 = 1.25^2$$
$$X^2 = 1.25^2 - Y^2$$
$$X = \sqrt{1.25^2 - Y^2}$$

If the cutter travels in the Y direction 0.5″, the X position can be calculated by substituting 0.5″ for Y.

$$X = \sqrt{1.25^2 - 0.5^2}$$
$$X = \sqrt{1.3125}$$
$$X = 1.146$$

**Figure 3-30.** This illustration shows how to use the Pythagorean theorem to calculate the cutter position as it creates a corner radius.

To start, we need to calculate the position of the cutter in Position 2 based on the center of the workpiece radius. We know that Y = 1.00″ and H = 2.25. Therefore, X can be calculated using Pythagorean's theorem.

$$X^2 + 1^2 = 2.25^2$$
$$X^2 = 2.25^2 - 1^2$$
$$X = \sqrt{2.25^2 - 1^2}$$
$$X = \sqrt{5.0625 - 1}$$
$$X = 2.0156$$

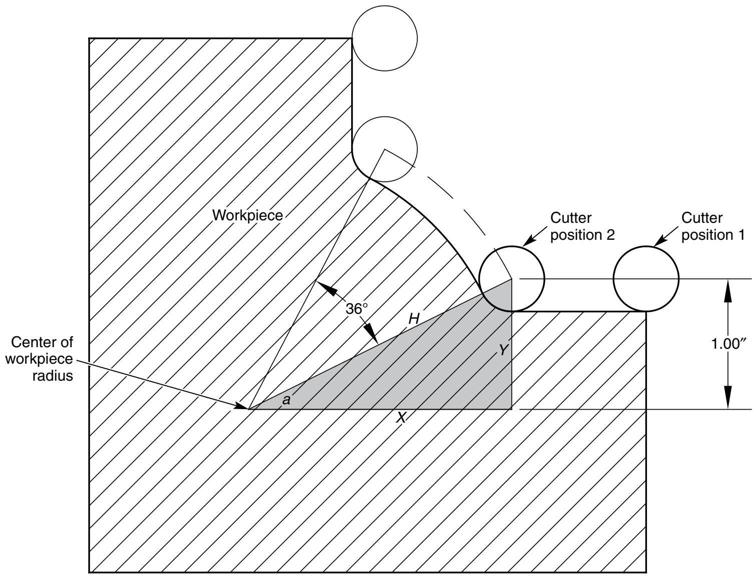



**Figure 3-31.** The Pythagorean theorem and trigonometric functions can be used together to plot the toolpath of a cutter.

We can use trigonometric functions to plot the positions of the cutter as it travels along the 2.00″ radius in 1° increments. To do this we have to find angle *a*.

$$\sin a = \frac{1}{2.25''}$$

$$a = \sin^{-1}(.4444)$$

$$a = 26°$$

Now we can calculate distance X for each degree that angle *a* increases until it reaches 62° (26° + 36°).

$$\sin 27 = \frac{Y}{2.25''}$$

$$\sin 27\ (2.25) = Y$$

$$1.021 = Y$$

$$\sin 28\ (2.25) = Y$$

$$1.056 = Y$$



# Summary

Various geometric principles relating to triangles, quadrilaterals, and circles are important to learn. Many of these principles are applied to obtain needed data for calculating tool locations. There are several propositions relating to angles that should be learned by an individual applying math to calculate tool location.

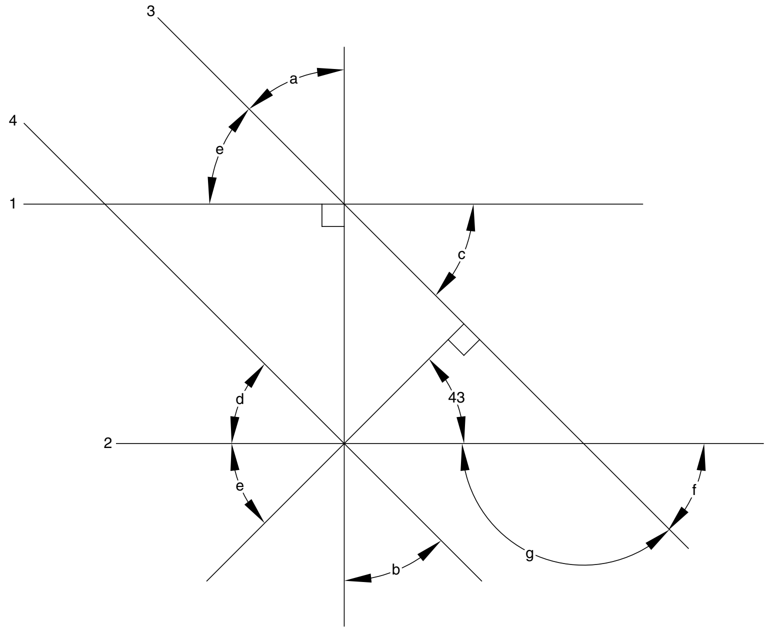
Using trigonometry to solve for the various parts of a triangle is an important concept. Tool location is often determined by using the trigonometric functions. There are six trigonometric functions and each is defined relative to the relationships between two sides of the right triangle.

## Chapter Review

*Answer the following questions. Write your answers on a separate sheet of paper.*

1. List the complementary angles for the following.
   a. 62°
   b. 41°
   c. 14°32′
2. List the supplementary angles for the following.
   a. 76°
   b. 167°
   c. 145°25′15″

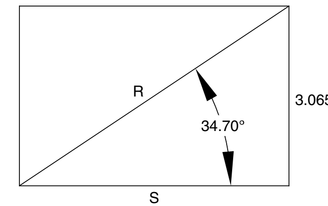
3. Determine the values of the angles shown in the figure below. State the propositions used in the problem. Lines 1 and 2 are parallel, and Lines 3 and 4 are parallel.



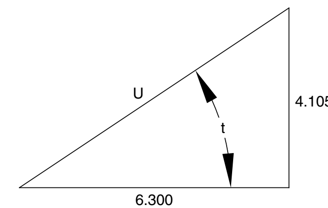

   a. Angle a
   b. Angle b
   c. Angle c
   d. Angle d
   e. Angle e
   f. Angle f
   g. Angle g
4. Using a table or calculator, determine the value of each function listed.
   a. Tan 33.15°
   b. Sin 23°
   c. Cos 26.6°
   d. Cot 41°



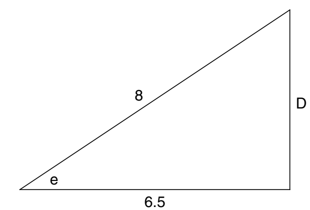
5. Using the triangle below, solve for R and S.




6. Using the triangle below, solve for U and t.

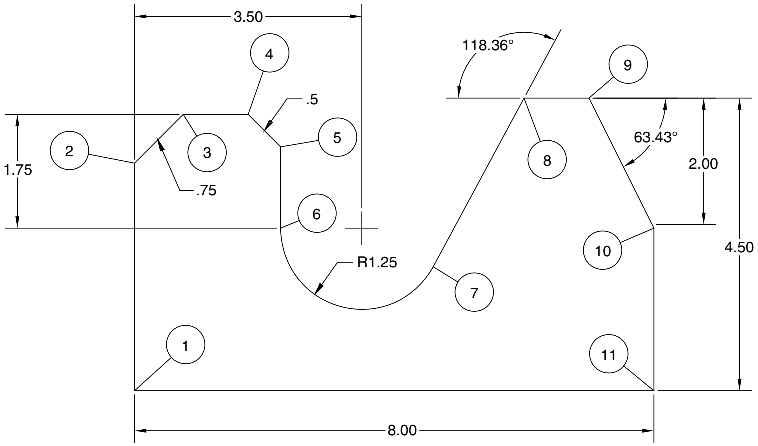



7. Using the triangle below, solve for D and e.



## Activities

1. Using the print, calculate the value of the positions identified by the numerals 1–11. It may be necessary to use the formulation of right triangles and trigonometry to calculate certain positions. Place the values determined into a table similar to the one shown.




| Location | X | Y |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |



| Right Triangle Formulas and Calculator Steps | | | |
|---|---|---|---|
| **To find...** | **...and you know...** | **...perform these calculator steps.** | **Formulas** |
| angle B | sides a & b | b ÷ a 2nd SIN | b/a = Sin B |
| angle B | sides a & c | c ÷ a 2nd COS | c/a = Cos B |
| angle B | sides b & c | b ÷ c 2nd TAN | b/c = Tan B |
| angle C | sides a & b | b ÷ a 2nd COS | b/a = Cos C |
| angle C | sides a & c | c ÷ a 2nd SIN | c/a = Sin C |
| angle C | sides b & c | c ÷ b 2nd TAN | c/b = Tan C |
| side a | sides b & c | b $x^2$ + c $x^2$ 2nd $x^2$ | $\sqrt{b^2 + c^2}$ |
| side a | side c & angle C | c ÷ ( C SIN ) = | $\frac{c}{\sin C}$ |
| side a | side c & angle B | c ÷ ( B COS ) = | $\frac{c}{\cos B}$ |
| side a | side b & angle B | b ÷ ( B SIN ) = | $\frac{b}{\sin B}$ |
| side a | side b & angle C | b ÷ ( C COS ) = | $\frac{b}{\cos C}$ |
| side b | sides a & c | a $x^2$ + c $x^2$ 2nd $x^2$ | $\sqrt{a^2 + c^2}$ |
| side b | side a & angle B | B SIN × a | a × Sin B |
| side b | side a & angle C | C COS × a | a × Cos C |
| side b | side c & angle B | B TAN × c | c × Tan B |
| side b | side c & angle C | c ÷ ( C TAN ) = | $\frac{c}{\tan C}$ |
| side c | sides a & b | a $x^2$ + b $x^2$ 2nd $x^2$ | $\sqrt{a^2 + b^2}$ |
| side c | side a & angle B | B COS × a | a × Cos B |
| side c | side a & angle C | C SIN × a | a × Sin C |
| side c | side b & angle B | b ÷ ( B TAN ) = | $\frac{b}{\tan B}$ |
| side c | side b & angle C | C TAN × b | b × Tan C |