Chapter 47

Phases of the Moon

By definition, the times of New Moon, First Quarter, Full Moon and Last Quarter are the times at which the excess of the apparent longitude of the Moon over the apparent longitude of the Sun is 0°, 90°, 180°, and 270°, respectively.

Hence, to calculate the instants of these lunar phases, it is necessary to calculate the apparent longitudes of the Moon and the Sun separately. (However, the effect of the nutation may be neglected here, since the nutation in longitude $\Delta\psi$ will not affect the *difference* between the longitudes of Moon and Sun.)

However, if no high accuracy is required, the instants of the lunar phases can be calculated by the method described in this Chapter. The expressions are based on Chapront's ELP-2000/82 theory for the Moon (with improved expressions for the arguments M, M', etc., as mentioned in Chapter 45), and on Bretagnon's and Francou's VSOP87 theory for the Sun. The resulting times will be expressed in Julian Ephemeris Days (JDE), hence in Dynamical Time.

The times of the *mean* phases of the Moon, already affected by the Sun's aberration and by the Moon's light-time, are given by

$$\begin{aligned} \text{JDE} = 2451\,550.09765 &+ 29.530\,588\,853\,k \\ &+ 0.000\,1337\,T^2 \\ &- 0.000\,000\,150\,T^3 \\ &+ 0.000\,000\,000\,73\,T^4 \end{aligned} \tag{47.1}$$

where an integer value of k gives a New Moon, an integer increased

by 0.25 gives a First Quarter,
by 0.50 gives a Full Moon,
by 0.75 gives a Last Quarter.

Any other value for k will give meaningless results ! The value $k = 0$ corresponds to the New Moon of 2000 January 6. Negative values of k give lunar phases before the year 2000.

For example,

+479.00 and −2793.00 correspond to a New Moon,
+479.25 and −2792.75 correspond to a First Quarter,
+479.50 and −2792.50 correspond to a Full Moon,
+479.75 and −2792.25 correspond to a Last Quarter.

An approximate value of k is given by

$$k \cong (\text{year} - 2000) \times 12.3685 \qquad (47.2)$$

where the 'year' should be taken with decimals, for example 1987.25 for the end of March 1987 (because this is 0.25 year since the beginning of the year 1987). The sign ≅ means "is approximately equal to".

Finally, in formula (47.1) T is the time in Julian centuries since the epoch 2000.0; it is obtained with a sufficient accuracy from

$$T = \frac{k}{1236.85} \qquad (47.3)$$

and hence is negative before the epoch 2000.0.

Calculate E by means of formula (45.6), and then the following angles, which are expressed in degrees and may be reduced to the interval 0-360 degrees and, if necessary, to radians before going further on.

Sun's mean anomaly at time JDE :

$$M = 2.5534 + 29.105\,356\,69\,k - 0.000\,0218\,T^2 - 0.000\,000\,11\,T^3 \qquad (47.4)$$

Moon's mean anomaly :

$$M' = 201.5643 + 385.816\,935\,28\,k + 0.010\,7438\,T^2 + 0.000\,012\,39\,T^3 - 0.000\,000\,058\,T^4 \qquad (47.5)$$

Moon's argument of latitude :

$$F = 160.7108 + 390.670\,502\,74\,k - 0.001\,6341\,T^2 - 0.000\,002\,27\,T^3 + 0.000\,000\,011\,T^4 \qquad (47.6)$$

Longitude of the ascending node of the lunar orbit :

$$\Omega = 124.7746 - 1.563\,755\,80\,k + 0.002\,0691\,T^2 + 0.000\,002\,15\,T^3 \qquad (47.7)$$

Planetary arguments :

$$A_1 = 299.77 + 0.107\,408\,k - 0.009\,173\,T^2$$
$$A_2 = 251.88 + 0.016\,321\,k$$
$$A_3 = 251.83 + 26.651\,886\,k$$
$$A_4 = 349.42 + 36.412\,478\,k$$
$$A_5 = 84.66 + 18.206\,239\,k$$
$$A_6 = 141.74 + 53.303\,771\,k$$
$$A_7 = 207.14 + 2.453\,732\,k$$
$$A_8 = 154.84 + 7.306\,860\,k$$
$$A_9 = 34.52 + 27.261\,239\,k$$
$$A_{10} = 207.19 + 0.121\,824\,k$$
$$A_{11} = 291.34 + 1.844\,379\,k$$
$$A_{12} = 161.72 + 24.198\,154\,k$$
$$A_{13} = 239.56 + 25.513\,099\,k$$
$$A_{14} = 331.55 + 3.592\,518\,k$$

To obtain the time of the *true* (apparent) phase, add the following corrections (in days) to the JDE obtained above.

New Moon	Full Moon	× sin
−0.40720	−0.40614	M'
+0.17241 × E	+0.17302 × E	M
+0.01608	+0.01614	$2M'$
+0.01039	+0.01043	$2F$
+0.00739 × E	+0.00734 × E	$M' - M$
−0.00514 × E	−0.00515 × E	$M' + M$
+0.00208 × E^2	+0.00209 × E^2	$2M$
−0.00111	−0.00111	$M' - 2F$
−0.00057	−0.00057	$M' + 2F$
+0.00056 × E	+0.00056 × E	$2M' + M$
−0.00042	−0.00042	$3M'$
+0.00042 × E	+0.00042 × E	$M + 2F$
+0.00038 × E	+0.00038 × E	$M - 2F$
−0.00024 × E	−0.00024 × E	$2M' - M$
−0.00017	−0.00017	Ω
−0.00007	−0.00007	$M' + 2M$
+0.00004	+0.00004	$2M' - 2F$
+0.00004	+0.00004	$3M$
+0.00003	+0.00003	$M' + M - 2F$
+0.00003	+0.00003	$2M' + 2F$
−0.00003	−0.00003	$M' + M + 2F$
+0.00003	+0.00003	$M' - M + 2F$
−0.00002	−0.00002	$M' - M - 2F$
−0.00002	−0.00002	$3M' + M$
+0.00002	+0.00002	$4M'$

First and Last Quarters

−0.62801		× sin M'
+0.17172	× E	M
−0.01183	× E	$M' + M$
+0.00862		$2M'$
+0.00804		$2F$
+0.00454	× E	$M' - M$
+0.00204	× E^2	$2M$
−0.00180		$M' - 2F$
−0.00070		$M' + 2F$
−0.00040		$3M'$
−0.00034	× E	$2M' - M$
+0.00032	× E	$M + 2F$
+0.00032	× E	$M - 2F$
−0.00028	× E^2	$M' + 2M$
+0.00027	× E	$2M' + M$
−0.00017		Ω
−0.00005		$M' - M - 2F$
+0.00004		$2M' + 2F$
−0.00004		$M' + M + 2F$
+0.00004		$M' - 2M$
+0.00003		$M' + M - 2F$
+0.00003		$3M$
+0.00002		$2M' - 2F$
+0.00002		$M' - M + 2F$
−0.00002		$3M' + M$

Calculate, for the Quarter phases only,

$$W = 0.00306 - 0.00038\,E\cos M + 0.00026\cos M' - 0.00002\cos(M' - M) + 0.00002\cos(M' + M) + 0.00002\cos 2F$$

Additional corrections :

for First Quarter : $+W$
for Last Quarter : $-W$

Additional corrections for all phases :

+0.000325 × sin	A_1	+0.000056 × sin	A_8
165	A_2	047	A_9
164	A_3	042	A_{10}
126	A_4	040	A_{11}
110	A_5	037	A_{12}
062	A_6	035	A_{13}
060	A_7	023	A_{14}

Example 47.a — Calculate the instant of the New Moon which took place in February 1977.

Mid-February 1977 corresponds to 1977.13, so we find by (47.2)

$$k \cong (1977.13 - 2000) \times 12.3685 = -282.87$$

whence $k = -283$, since k should be an integer for the New Moon phase. Then, by formula (47.3), $T = -0.22881$, and then formula (47.1) gives

$$\text{JDE} = 2443\,192.94101$$

With $k = -283$ and $T = -0.22881$, we further find

$$E = 1.000\,5753$$
$$M = -8\,234^\circ.2625 = 45^\circ.7375$$
$$M' = -108\,984^\circ.6278 = 95^\circ.3722$$
$$F = -110\,399^\circ.0416 = 120^\circ.9584$$
$$\Omega = 567^\circ.3176 = 207^\circ.3176$$

The sum of the first group of periodic terms (for New Moon) is −0.28916, that of the 14 additional corrections is −0.00068. Consequently, the time of the true New Moon is

$$\text{JDE} = 2443\,192.94101 - 0.28916 - 0.00068 = 2443\,192.65117,$$

which corresponds to 1977 February 18.15117 TD
= 1977 February 18, at $3^h37^m41^s$ TD.

The correct value, calculated by means of the ELP-2000/82 theory, is $3^h37^m40^s$ TD.

In February 1977, the quantity ΔT = TD − UT was equal to 48 seconds. Hence, the New Moon of 1977 February 18 occurred at 3^h37^m Universal Time. See also Example 9.a, page 74.

Example 47.b — Calculate the time of the first Last Quarter of A.D. 2044.

For 'year' = 2044, formula (47.2) gives $k \cong +544.21$, so we shall use the value $k = +544.75$.

Then, by formula (47.1), JDE = 2467 636.88595.

Sum of the first group of periodic terms (for Last Quarter) = −0.39153.

Additional correction for Last Quarter = $-W$ = −0.00251.

Sum of additional 14 corrections = −0.00007.

Consequently, the time of the Last Quarter is

$$2467\,636.88595 - 0.39153 - 0.00251 - 0.00007 = 2467\,636.49184$$

which corresponds to 2044 January 21, at $23^h48^m15^s$ TD.